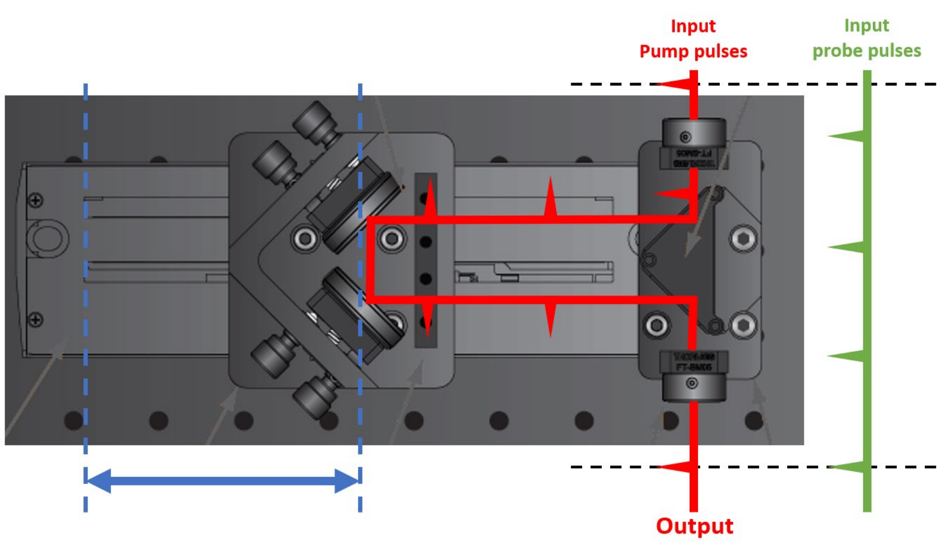
Exercise 11.1: Transient absorption

Explain briefly how an optical delay line works and calculate the length that is necessary in an ultrafast transient absorption experiment to measure a signal that completely decays in 20 ns. Is this a realistic experiment? What alternative experiment is possible when measuring signals on a similar timescale?

Exercise 11.1 solution.

An optical delay line is used to reconstruct the signal decay in time on femtosecond-picosecond timescales. One needs a long-enough delay line for delaying the pump by 20 ns with respect to the probe. Given the speed of light (approx. 3E8 m/s) one finds an optical delay of $d = c \cdot \Delta t = 6\ m$. Because optical lines usually work as a double path, this would correspond to roughly a 3 m optical delay line. In practice, this would mean that a distance of 3 m would be needed in the lab to build an optical delay line. From a practical point of view, to resolve signals in the nanosecond timescale, using an oscilloscope is easier. There are also a few (more expensive) options to detect electronically signals below the nanosecond, but a simple oscilloscope will routinely provide a few ns resolution.



Exercise 11.2: Quantum size effects

Estimate the temperature at which quantum size effects would be important for a semiconductor layer of thickness 1 μm if the effective mass of the electrons is 0.1 m_0. (m_0 = 9.11 x 10^{-31} kg)

Exercise 11.2 solution.

Here we restrict an electron's position in a semiconductor layer of $\Delta x = 1\ \mu\text{m}$. This means that we know with more accuracy (Δx) the position of the electron. According to the uncertainty principle, $\Delta x \Delta p_x \leq \hbar$ so $\Delta p_x \leq \hbar/\Delta x$.

The confinement energy is given by $E_{conf} = \frac{(\Delta p)^2}{2m}$. Using the uncertainty principle, one finds that

$$E_{conf} \geqq \frac{\hbar^2}{2m(\Delta x)^2}$$

One can consider that the particle is confined when its confinement energy is larger than the thermal kinetic energy, $k_B T/2$. Thus, we are looking for T such that $\frac{\hbar^2}{2m(\Delta x)^2} > k_B T/2$.

This results in the following criterium: $T < \frac{\hbar^2}{k_B m(\Delta x)^2}$, with $m = 0.1\ m_o$, $\hbar = h/2\pi$

One finds that the temperature under which confinement occurs is 0.01 K.

This calculation shows that the confinement energy is generally negligible for a semiconductor quantum well of 1 μm.

Exercise 11.3: Absorption in quantum wells

Using the data for GaAs listed the table below, estimate the difference in the wavelength of the absorption edge of a 20 nm GaAs quantum well and bulk GaAs at 300 K.

Table D.2 Band structure parameters for selected direct gap III–V semiconductors with the zinc-blende structure. The parameters listed refer to the four-band model shown in Fig. 3.5. E_g: band gap; Δ: spin–orbit splitting; m_e^*: electron effective mass; m_{hh}^*: heavy-hole effective mass; m_{lh}^*: light-hole effective mass; m_{so}^*: split-off hole effective mass. The effective masses are expressed in units of the free electron mass m_0. After Madelung (1996) and Madelung (1982).

Crystal	E_g (eV) (0 K)	E_g (eV) (300 K)	Δ (eV)	m_e^*	m_{hh}^*	m_{lh}^*	m_{so}^*
GaAs	1.519	1.424	0.34	0.067	0.5	0.08	0.15
GaSb	0.81	0.75	0.76	0.041	0.28	0.05	0.14
InP	1.42	1.34	0.11	0.077	0.6	0.12	0.12
InAs	0.42	0.35	0.38	0.022	0.4	0.026	0.14
InSb	0.24	0.18	0.85	0.014	0.4	0.016	0.47

Exercise 11.3 solution.

The optical transition in a quantum well is not equal to the bandgap, because of the confinement energies that come in addition, for the holes and electrons. For the holes, E_{hh1} corresponds to the ground-state of the valence band ($n = 1$ heavy-hole level), and for the electrons, E_{e1} corresponds to the lowest conduction band state ($n = 1$ electron level). The absorption edge of a quantum well occurs at $E_g + E_{hh1} + E_{e1}$.

As shown in the lecture, using the infinite potential well model, one can approximate the confinement energy of the electrons and holes by $E_{e/h} = \frac{\hbar^2 n^2 \pi^2}{2m^* (\Delta x)^2}$ where m^* denotes the effective mass. We find that $E_{hh1} = 2meV$ and $E_{e1} = 14\, meV$.

These energies are small compared to typical quantum well barrier heights, and so the infinite well approximation should be reasonably accurate. The band edge therefore shifts from 1.424 eV to $(1.424 + 0.002 + 0.014) = 1.440\, eV$. This corresponds to a blue shift of 10 nm.